Dynamic Characteristics of an Airplane

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Aircraft Equations of Motion

An understanding of the dynamic characteristics of an airplane is important in assessing its handling or flying qualities as well as for designing autopilots. The models are developed from the equations of motion which contain the aerodynamic derivatives. Because these stability derivatives are a function of the geometric and aerodynamic characteristics of the airplane, designers have some control over the longitudinal dynamics by their selection. The linearized longitudinal equations developed in this section are simple, ordinary linear differential equations with constant coefficients. The coefficients are made up of the aerodynamic stability derivatives, mass and inertia characteristics of the airplane. Before developing the equations of motion, it is important to review the axis system.

![Figure 1: The Axis System](image)

The rigid body equations of motion are obtained from Newton’s second law, which states that the summation of all external forces acting on a body is equal to the time rate of change of the momentum of the body.

$$\sum F = \frac{d}{dt}(mv) \quad \sum M = \frac{d}{dt}H$$  \hspace{1cm} (1)

The vector equations can be rewritten in scalar form and then consist of three force equations and three moment equations.

Rewriting these vector equations and using a small disturbance theory and gravitational and thrust forces acting on the airplane (see Robert C. Nelson - Flight Stability and Automatic Control [2]) will yield the equations of motion.

The Longitudinal equations are:

\[
\begin{align*}
\left(\frac{d}{dt} - X_u\right)\Delta u + X_u\Delta w + (g \cos \theta_0)\Delta \theta &= X_{\delta_e}\Delta \delta_e + X_{\delta_T}\Delta \delta_T \\
-Z_u\Delta u + \left[(1 - Z_\omega)\frac{d}{dt} - Z_\omega\right]\Delta w - \left[(u_0 + Z_q)\frac{d}{dt} - g \sin \theta_0\right]\Delta \theta &= Z_{\delta_e}\Delta \delta_e + Z_{\delta_T}\Delta \delta_T \\
-M_u\Delta u - \left(M_u\frac{d}{dt} + M_w\right)\Delta w + \left(\frac{d^2}{dt^2} - M_\omega\frac{d}{dt}\right)\Delta \theta &= M_{\delta_e}\Delta \delta_e + M_{\delta_T}\Delta \delta_T
\end{align*}
\]  \hspace{1cm} (2)
TABLE 1
Summary of longitudinal derivatives

\[
\begin{align*}
X_u &= \frac{-(C_{D_\alpha} + 2C_{D_\theta} QS)}{m u_0} \\
X_w &= \frac{-(C_{D_\alpha} + 2C_{L_\theta} QS)}{m u_0} \\
Z_u &= \frac{-(C_{L_\alpha} + 2C_{L_\theta} QS)}{m u_0} \\
Z_w &= \frac{-(C_{L_\alpha} + 2C_{L_\theta} QS)}{m u_0} \\
Z_q &= -C_{Z_q} \frac{\dot{\alpha}}{2u_0} QS/m \\
Z_{\delta e} &= -C_{Z_{\delta e}} QS/m \\
M_u &= C_{m_u} \frac{QS\dot{c}}{u_0 I_y} \\
M_w &= C_{m_w} \frac{QS\dot{c}}{u_0 I_y} \\
M_q &= C_{m_q} \frac{\dot{c}}{2u_0} QS/m \\
M_{\delta e} &= C_{m_{\delta e}} QS\dot{c}/I_y
\end{align*}
\]

where \(C_{D_\alpha}\) is a change in the drag coefficient with forward speed, \(C_{D_\theta}\) is a reference drag coefficient, \(C_{D_\alpha}\) is a change in the drag coefficient with the angle of attack, \(C_{Z_q}\) and \(C_{m_q}\) are the stability coefficients representing the change in the Z force and pitching moment coefficients with respect to the pitching velocity \(q\) and \(C_{L_\alpha}\) is a change in lift coefficient with the Mach number. \(C_{m_u}\) is the change in pitching moment due to variations in the forward speed that depends on the Mach number and also is affected by the elastic properties of the airframe.
The Lateral equations are:

\[
\begin{align*}
\left( \frac{d}{dt} - Y_v \right) \Delta v - Y_p \Delta p + (u_0 - Y_r) \Delta r - g \cos \theta_0 \Delta \Phi &= Y_\delta, \Delta \delta_r \\
-L_v \Delta v + \left( \frac{d}{dt} - L_p \right) \Delta p - \left( \frac{I_{yz}}{I_x} \frac{d}{dt} + L_r \right) \Delta r &= L_\delta, \Delta \delta_a + L_\delta_r, \Delta \delta_r \\
-N_\alpha \Delta v - \left( \frac{I_{xz}}{I_z} \frac{d}{dt} + N_p \right) \Delta p + (\frac{d}{dt} - N_r) \Delta r &= N_\delta, \Delta \delta_a + N_\delta_r, \Delta \delta_r
\end{align*}
\]

(3)

TABLE 2
Summary of lateral directional derivates

<table>
<thead>
<tr>
<th>Term</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_\beta$</td>
<td>$\frac{Q_{sb} C_{\alpha \beta}}{I_s}$</td>
</tr>
<tr>
<td>$L_\beta$</td>
<td>$\frac{Q_{sb} C_{\beta \theta}}{I_s}$</td>
</tr>
<tr>
<td>$Y_\beta$</td>
<td>$\frac{Q_{sc} \gamma}{m}$</td>
</tr>
<tr>
<td>$N_\rho$</td>
<td>$\frac{Q_{sb}^2 C_{\alpha \rho}}{2 I_s u_0}$</td>
</tr>
<tr>
<td>$L_\rho$</td>
<td>$\frac{Q_{sb}^2 C_{\rho \theta}}{2 I_s u_0}$</td>
</tr>
<tr>
<td>$Y_\rho$</td>
<td></td>
</tr>
<tr>
<td>$N_\delta_a$</td>
<td>$\frac{Q_{sb} C_{\alpha \delta_a}}{I_s}$</td>
</tr>
<tr>
<td>$L_\delta_a$</td>
<td>$\frac{Q_{sb} C_{\delta_a \theta}}{I_s}$</td>
</tr>
<tr>
<td>$Y_\delta_a$</td>
<td>$\frac{Q_{sc} \delta_a}{m}$</td>
</tr>
<tr>
<td>$N_\delta_r$</td>
<td>$\frac{Q_{sb} C_{\alpha \delta_r}}{I_s}$</td>
</tr>
<tr>
<td>$L_\delta_r$</td>
<td>$\frac{Q_{sb} C_{\delta_r \theta}}{I_s}$</td>
</tr>
<tr>
<td>$Y_\delta_r$</td>
<td>$\frac{Q_{sc} \delta_r}{m}$</td>
</tr>
</tbody>
</table>

The lateral-directional equations of motion consist of the side force, rolling moment and yawing moment. The stick fixed lateral motion of an airplane disturbed from its equilibrium state is a complicated combination of rolling, yawing and sideslipping motions. An airplane produces both yawing and rolling moments due to the sideslip angle. This interaction between the roll and the yaw produces the coupled motion. Three potential lateral dynamic instabilities are of interest to the airplane designer: directional divergence, spiral divergence, and so-called Dutch roll oscillation. Thus a lateral motion is more difficult from designer's point and the designer has to consider more objections when designing an airplane.

State Variable Representation of the Equations of Longitudinal Motion

The state space model for longitudinal motion is:

\[
\dot{x} = Ax + B\eta
\]

(4)

where $x$ is a state vector, $\eta$ is a control vector and the matrices $A$ and $B$ contain the aircraft's dimensional stability derivates. Rewriting equations (2) in the state-space form yields
The model of the airplane of longitudinal motion is Multiple-Input Multiple-Output system with two inputs (thrust engine and elevator) and four outputs (velocity, angle of attack, pitch angle and pitch rate).

The longitudinal motion of an airplane disturbed from its equilibrium flight condition is characterized by two oscillatory modes of motion. One is lightly damped and has a long period. This motion is called the long period motion. It is excited by thrust engine by constant deflection of the elevator and characterized by changes in pitch attitude, altitude and velocity at a nearly constant angle of attack. It is described by the two following equations:

\[
\frac{d}{dt} \Delta u - Z_u \cdot \Delta u + \frac{g}{u_0} = X_{\delta_T} \cdot \Delta \theta
\]

\[
-Z_u \cdot \Delta u - \frac{d}{dt} \Delta \theta = 0
\]

The second motion is heavily damped and has a very short period; it is called the short-period mode. It is characterized by motion excited by deflection of the elevator at a constant thrust engine. That’s the time period when the angle of attack is changing while the velocity is nearly constant. The next following equations describe this motion:

\[
-M_w \cdot \frac{d}{dt} \Delta \alpha - M_w \cdot \Delta \alpha + \frac{d^2}{dt^2} \Delta \theta - M_q \cdot \frac{d}{dt} \Delta \theta = M_{\delta_e} \cdot \delta_e
\]

\[
\frac{d}{dt} \Delta \alpha - Z_w \cdot \Delta \alpha - \frac{d}{dt} \Delta \theta = 0
\]

Of the two characteristic modes, the short-period mode is the more important one. If this mode has a high frequency and is heavily damped, then the airplane will respond rapidly to an elevator input without any undesirable overshoot. When the short-period mode is lightly damped or has a relatively low frequency, the airplane will be difficult to control and in some cases may even be dangerous to fly.

The phugoid or long-period mode occurs so slowly that the pilot can easily negate the disturbance by small control movements. Even though the pilot can correct easily for the phugoid mode it would be extremely fatiguing if the damping was too low.

The flying qualities of an airplane are related to the stability and control characteristics and can be defined as those stability and control characteristics important in forming the pilot’s impression of the airplane. The pilot forms a subjective opinion about the easy or difficulty of controlling the airplane in steady and maneuvering flight.

The short- and long-period damping ratios and undamped natural frequencies influence the pilot’s opinion. Although we can calculate these qualities, the question that needs to be answered is what values should \( \zeta \) and \( \omega_n \) take so that the pilot finds the airplane easy to
fly. These data are specified in MIL-STD-1797B [1].

Of course, flight qualities are depended on the type of aircraft and the flight phase. Aircraft are classified according to size and maneuverability as shown in table (3) and the flight phases are in table (4).

The flying qualities are specified in terms of three levels:

- **Level 1**: Flying qualities clearly adequate for the mission flight phase
- **Level 2**: Flying qualities adequate to accomplish the mission phase but with some increase in pilot workload and/or degradation in mission effectiveness or both
- **Level 3**: Flying qualities such that the airplane can be controlled safely but pilot workload is excessive and/or mission effectiveness is inadequate or both. Category A flight phase can be terminated safely and Category B and C flight phases can be completed.

### TABLE 3
**Classification of airplanes**

<table>
<thead>
<tr>
<th>Class</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class I</td>
<td>Small, light airplanes, such as light utility, primary trainer, and light observation aircraft</td>
</tr>
<tr>
<td>Class II</td>
<td>Medium-weight, low-to-medium maneuverability airplanes, such as heavy utility/search and rescue, light or medium transport/cargo/tanker...</td>
</tr>
<tr>
<td>Class III</td>
<td>Large, heavy, low-to-medium maneuverability airplanes, such as heavy transport/cargo/tanker, heavy bomber and trainer for Class III.</td>
</tr>
<tr>
<td>Class IV</td>
<td>High-maneuverability airplanes, such as fighter/interceptor, attack, tactical reconnaissance, observation and trainer for Class IV.</td>
</tr>
</tbody>
</table>

### TABLE 4
**Flight phase categories**

#### Nonterminal flight phase

- **Category A**: Nonterminal flight phase, that require rapid maneuvering, precision tracking, or precise flight-path control. Included in the category are air-to-air combat ground attack, weapon delivery/launch, aerial recovery, reconnaissance...
- **Category B**: Nonterminal flight phases that are normally accomplished using gradual maneuvers and without precision tracking, although accurate flight-path control may be required. Included in the category are climb, cruise, loiter, in-flight refueling (tanker), descent, emergency descent, deceleration, and aerial delivery.

#### Terminal flight phase

- **Category C**: Terminal flight phases are normally accomplished using gradual maneuvers and usually require accurate flight-path control. Included in this category are takeoff, catapult takeoff, approach, wave-off/go-around and landing.

Next table (5) shows us a summary of the longitudinal specifications for the phugoid and short-period motions that is valid for all classes of aircraft.

To improve the flying qualities of an airplane, the designer needs to provide more short-period damping. This could be accomplished by increasing the tail area or the tail moment arm. Such geometric changes would increase the stability coefficients $C_{m_{\alpha}}$, $C_{m_{q}}$ and $C_{m_{\dot{\alpha}}}$. Unfortunately this cannot be accomplished without a penalty in flight performance.
TABLE 5

Recommended Short Period Requirements for Class III

<table>
<thead>
<tr>
<th>Level</th>
<th>Min $\zeta_{sp}$</th>
<th>Max $\zeta_{sp}$</th>
<th>$\tau_{\theta}$, sec</th>
<th>Min $\omega_{sp}/I_{\theta_0}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.30</td>
<td>2.00</td>
<td>0.10</td>
<td>1.00</td>
</tr>
<tr>
<td>2</td>
<td>0.20</td>
<td>2.00</td>
<td>0.20</td>
<td>0.60</td>
</tr>
<tr>
<td>3</td>
<td>-</td>
<td>-</td>
<td>0.25</td>
<td>-</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Category C</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
</tbody>
</table>

State Variable Representation of the Equations of Lateral Motion

Lateral equations (3) can be rearranged into the state-space form:

\[
\begin{bmatrix}
\Delta \dot{\beta} \\
\Delta \dot{\rho} \\
\Delta \dot{r} \\
\Delta \phi
\end{bmatrix} =
\begin{bmatrix}
\frac{Y_\alpha}{U_0} & \frac{Y_\rho}{U_0} & -1 & \frac{g \cos \theta_0}{U_0} \\
L_\beta & L_p & L_r & 0 \\
N_\beta & N_p & N_r & 0 \\
0 & 1 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\Delta \beta \\
\Delta \rho \\
\Delta r \\
\Delta \phi
\end{bmatrix} +
\begin{bmatrix}
0 \\
L_{\delta_a} & L_{\delta_r} \\
N_{\delta_a} & N_{\delta_r}
\end{bmatrix}
\begin{bmatrix}
\Delta \delta_a \\
\Delta \delta_r
\end{bmatrix}
\]

(10)

This model is a MIMO system with two inputs, aileron and rudder and four outputs, sideslip angle, angular velocities $p, r$ about axes $X$ and $Z$ and Euler angle $\phi$ which is the roll angle. The variables are shown in figures (1) and (2).

In general, we will find the roots to the lateral-directional characteristic equation to be composed of two real roots and a pair of complex roots. The roots will be such that the airplane response can be characterized by the following motions:

1. A slowly convergent or divergent motion, called the spiral mode.
2. A highly convergent motion, called the rolling mode.
3. A lightly damped oscillatory motion having a low frequency, called the Dutch roll mode.

The two most used approximations of the lateral motion are Dutch Roll approximation and Roll approximation.

Dutch Roll Approximation is characterized by motion excited by deflection of the rudder at still ailerons. We assume that part of solution without the rolling moment. This motion is described by the following equations:

\[
\Delta \dot{\beta} + \Delta \beta \frac{Y_\beta}{U_0} - \Delta r = 0
\]

(11)

\[
\Delta \beta N_\beta + \Delta r + \Delta r N_r = N_{\delta_a}, \delta_r
\]

(12)
From these equations we can determine the undamped natural frequency and the damping ratio as following:

\[
\omega_{n,DR} = \sqrt{\frac{Y \cdot N_R - N_\beta \cdot Y_R + u_0 \cdot N_\beta}{u_0}}
\] (13)

\[
\zeta_{DR} = -\frac{1}{2\omega_{n,DR} \left( Y_\beta + u_0 \cdot N_R \right) u_0}
\] (14)

The approximations developed in this section give, at best, only a rough estimate of the spiral and Dutch roll modes. The approximate formulas should, therefore, be used with caution. The reason for the poor agreement between the approximate and exact solutions is that the Dutch roll motion is truly a three-degree-of-freedom motion with strong coupling between the equations.

**Roll Approximation** represents turning round a longitudinal axis excited by deflection of ailerons at still rudder. We assume such a part of solution without turning motion \(N\) and side force \(Y\). This motion is described by one differential equation:

\[
\Delta \dot{p} + \Delta p L_p = L_{\delta_a} \delta_a
\] (15)

To determine whether the airplane has acceptable flying characteristics the designer needs to know what dynamic characteristics are considered to be favourable by the pilots who will fly the airplane. This information can be found in MIL-SD-1797B [1]. The following tables show required parameters for handling the flying qualities

<table>
<thead>
<tr>
<th>TABLE 6</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Recommended Roll-Mode Time Constant for Class III</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Level</th>
<th>Max (T_r), sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.4</td>
</tr>
<tr>
<td>2</td>
<td>3.0</td>
</tr>
<tr>
<td>3</td>
<td>10.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TABLE 7</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Recommended Dutch Roll Frequency and Damping for Class III</strong></td>
</tr>
</tbody>
</table>

| Category B |

<table>
<thead>
<tr>
<th>Level</th>
<th>Min (\zeta_d)</th>
<th>Min (\zeta_d \omega_d)</th>
<th>Min (\omega_d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.08</td>
<td>0.15</td>
<td>0.4</td>
</tr>
<tr>
<td>2</td>
<td>0.02</td>
<td>0.05</td>
<td>0.4</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>-</td>
<td>0.4</td>
</tr>
</tbody>
</table>

| Category C |

<table>
<thead>
<tr>
<th>Level</th>
<th>Min (\zeta_d)</th>
<th>Min (\omega_d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.08</td>
<td>0.10</td>
</tr>
<tr>
<td>2</td>
<td>0.02</td>
<td>0.05</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>-</td>
</tr>
</tbody>
</table>

**Transport aircraft: Boeing 747-400**

The incremental state-variable description for both longitudinal and lateral motions was developed in the previous sections. In this section we will specialize the results to the case
of Boeing 747-400 for certain altitude and Mach number. All derivates for longitudinal motion are presented in table (1) and derivates for lateral motion are in table (2).

All the derivates for our airplane are performed at \( M = 0.9 \) and \( H = 40\,000\text{ft} \). To obtain the velocity of the aircraft we need to know the speed of sound in the altitude 40 000ft.

The speed of sound can be obtained from the following:

\[
a = \sqrt{\gamma \cdot R \cdot T}
\]

where \( \gamma = 1.4 \) is the ratio of specific heats \( C_p/C_V \) for the isentropic flow, \( R = 287.1\,J/kgK \) is the molecular gass constant and \( T \) notes the temperature at 40 000ft. From atmospheric tables we will get \( T/T_0 = 0.7517 \), and \( \rho/\rho_0 = 0.2536 \).

\[
T_0 = 288.15\,K \quad T = 0.7517 \cdot 288.15 = 216\,K
\]

\[
\rho_0 = 1.225\,kg/m^3 \quad \rho = 0.2536 \cdot 1.225 = 0.3107\,kg/m^3
\]

\[
a = 296\,m/s \quad u_0 = M \cdot a = 265\,m/s = 869\,ft/s
\]

Before computing other derivates I will mention the English units:

- \( 1\,ft = 0.3048\,m \)
- \( 1\,lb = 0.454\,kg \)
- \( 1\,kg = 2.2026\,lb \)
- \( 14.62\,kg = 1\,slug \)
- \( 1\,m = 3.2508\,ft \)
- \( 1\,m^2 = 35.3147\,ft^3 \)
- \( 1\,kg/m^3 = 0.002377\,slug/ft^3 \)

Mass characteristics and reference geometry:

\[
m = 19769.32\,slugs
\]

\[
I_x = 18.2 \cdot 10^6\,Slug \cdot ft^2 \quad I_y = 33.1 \cdot 10^6\,Slug \cdot ft^2
\]

\[
I_z = 49.7 \cdot 10^6\,Slug \cdot ft^2 \quad I_{zz} = 0.97 \cdot 10^6\,Slug \cdot ft^2
\]

\[
S = 5500\,ft^2 \quad b = 195.68\,ft \quad c = 27.31\,ft
\]

### Longitudinal motion

**TABLE 8**

<table>
<thead>
<tr>
<th>( C_{La} )</th>
<th>( C_{Da} )</th>
<th>( C_{La} )</th>
<th>( C_{Da} )</th>
<th>( C_{n} )</th>
<th>( C_{La} )</th>
<th>( C_{m} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.042</td>
<td>5.5</td>
<td>0.47</td>
<td>-1.6</td>
<td>0.006</td>
<td>-9.0</td>
</tr>
<tr>
<td>( C_{La} )</td>
<td>( C_{n} )</td>
<td>( C_{La} )</td>
<td>( C_{n} )</td>
<td>( C_{La} )</td>
<td>( C_{n} )</td>
<td>( C_{La} )</td>
</tr>
<tr>
<td>6.58</td>
<td>-25</td>
<td>0.2</td>
<td>0.25</td>
<td>-0.1</td>
<td>0.3</td>
<td>-1.2</td>
</tr>
</tbody>
</table>

We still need another four derivates which we will obtain from the following equations:

\[
C_{Da} = M \cdot \frac{\partial C_D}{\partial C_M}, \quad \text{where} \quad \frac{\partial C_D}{\partial C_M} = C_{DM}
\]

\[
C_{m} = M \cdot \frac{\partial C_m}{\partial C_M}, \quad \text{where} \quad \frac{\partial C_m}{\partial C_M} = C_{mM}
\]

\[
C_L = \frac{M^2}{1 - M^2} C_{lo}
\]

\[
C_{Z_{e}} = -C_{L_{e}}
\]
Evaluating the equations will yield \( C_{D_u} = 0.225, C_{m_w} = -0.09, C_{l_w} = 2.1316 \). Substituting the numerical values of the stability derivatives into equation in the table (1), and then into (5), we can obtain the state-space system:

\[
\begin{bmatrix}
\Delta \dot{u} \\
\Delta \dot{w} \\
\Delta \dot{\theta} \\
\Delta \dot{q}
\end{bmatrix} = \begin{bmatrix}
-0.0225 & 0.0022 & -32.3819 & 0 \\
-0.2282 & -0.4038 & 0 & 869 \\
0 & 0 & 0 & 1 \\
-0.0001 & -0.0018 & 0 & -0.5518
\end{bmatrix} \begin{bmatrix}
\Delta u \\
\Delta w \\
\Delta \theta \\
\Delta q
\end{bmatrix} + \begin{bmatrix}
0.5000 & 0 \\
0 & -0.0219 \\
0 & 0 \\
0 & -1.2394
\end{bmatrix} \begin{bmatrix}
\Delta \delta_T \\
\Delta \delta_e
\end{bmatrix}
\]

The transfer functions can be obtained e.g. by using Matlab function

\[
[numiu, den] = ss(A, B, C, D, iu)
\]

\[
sys1 = tf(numiu(1,:), den)
\]

where matrixes \( A, B \) are in the state-space model and matrixes \( C, D \) are:

\[
C = \text{eye}(4), \quad D = \text{zero}(4, 2)
\]

The characteristic equation for the longitudinal motion is:

\[
\lambda^4 + 0.9782\lambda^3 + 1.838\lambda^2 + 0.03908\lambda + 0.01265 = 0
\]

Using Matlab function \( \text{damp} \) will yield the eigenvalues of the matrix \( A \):

\[
\lambda_{1,2} = -0.0573 \pm 0.186i \quad (\text{phugoig})
\]

\[
\lambda_{3,4} = -0.546 \pm 0.188i \quad (\text{short period})
\]

The damping ratio and the natural frequency are as following:

\[
\zeta_{n_p} = 0.107 \quad \omega_p = 0.0835 \text{ rad/s}
\]

\[
\zeta_{n_sp} = 0.356 \quad \omega_{sp} = 1.35 \text{ rad/s}
\]

We can obtain the high pitch attitude zero e.g.

\[
z = \text{zero}(sys3), \quad \text{abs}(1/z(1))
\]

\( T_{\theta_2} = 2.4846 \)

<table>
<thead>
<tr>
<th>( \zeta_{sp} )</th>
<th>( \omega_{sp} \cdot T_{\theta_2} )</th>
<th>Category</th>
<th>Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.356</td>
<td>3.4281</td>
<td>B</td>
<td>1</td>
</tr>
</tbody>
</table>
Lateral motion

<table>
<thead>
<tr>
<th>TABLE 10</th>
<th>Lateral Derivates</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{\beta}$</td>
<td>$C_{n_{\beta}}$</td>
</tr>
<tr>
<td>-0.85</td>
<td>-0.1</td>
</tr>
</tbody>
</table>

To determine the characteristics of the lateral motion of the model we have to get the state space system. Evaluating parameters in the table (2) will yield:

\[
\begin{bmatrix}
\Delta \beta \\
\Delta p \\
\Delta r \\
\Delta \phi \\
\end{bmatrix} = 
\begin{bmatrix}
-0.0619 & 0 & -1.0000 & 0.0373 \\
-1.3459 & -0.4546 & 0.3031 & 0 \\
0.9978 & 0.1123 & -0.1826 & 0 \\
0 & 1.0000 & 0 & 0 \\
\end{bmatrix} 
\begin{bmatrix}
\Delta \beta \\
\Delta p \\
\Delta r \\
\Delta \phi \\
\end{bmatrix}
\]

\[
+ \begin{bmatrix}
0 & 4.7490 \\
0.1884 & 0.0673 \\
0.0150 & -0.4490 \\
0 & 0 \\
\end{bmatrix} \begin{bmatrix}
\Delta \delta a \\
\Delta \delta r \\
\end{bmatrix}
\]

We will get the characteristic equation and its eigenvalues in the same way as in the previous.

\[
\lambda^4 + 3.496\lambda^3 + 7.199\lambda^2 + 8.19\lambda - 0.05282 = 0
\]

<table>
<thead>
<tr>
<th>TABLE 11</th>
<th>The lateral eigenvalues</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_i$ &amp; $\zeta$ &amp; $\omega$</td>
<td></td>
</tr>
<tr>
<td>Spiral root &amp; 0.00584 &amp; -1.00</td>
<td></td>
</tr>
<tr>
<td>Dutch roll roots &amp; -0.165 ± 0.969i &amp; 0.168</td>
<td></td>
</tr>
<tr>
<td>Roll root &amp; -0.374 &amp; 1.00</td>
<td></td>
</tr>
</tbody>
</table>

As we can see one all the poles correspond with a typical description of lateral motion apart from the roll root that I would expect to be at least ten times greater.

<table>
<thead>
<tr>
<th>TABLE 12</th>
<th>Handling qualities predictions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_r$</td>
<td>$\zeta_d$</td>
</tr>
<tr>
<td>2.673</td>
<td>0.168</td>
</tr>
</tbody>
</table>

Summary

The equations of motion of both longitudinal and lateral motion were presented and their state-space models were created and applied to two airplane models: Boeing 747-400 and L410. The characteristic equations and the eigenvalues of both the longitudinal and the lateral models were found. From their results was shown that the airplane model Boeing 747 has its flying qualities of level 1 for longitudinal motion and level 2 for lateral motion.
Three-view drawing of the Boeing 747-400 jet transport
Bibliography
