Close Loop System Identification of Dynamic Parameters of an Airplane

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October 2004
Chapter 1

Closed Loop System Identification

Identifiability of Closed Loop Systems

Sometimes it is important to identify systems with a dynamic feedback. It may not be possible to remove the system and identify it as an open loop system because of security reasons etc. However, there are several ways how to identify closed loop systems. I will mention two of them here.

**Indirect method based on additive test signal** is the one giving better results. Assuming exact knowledge of the controller, the forward system can be derived from the identified model. Transfer function of regulator will be

\[ G_R(z) = \frac{Q(z)}{R(z)} = \frac{q_\nu z^\nu + \cdots + q_1 z + q_0}{r_\mu z^\mu + \cdots + r_1 z + r_0} \]  

and transfer function of a system

\[ G_S(z) = \frac{B(z)}{A(z)} = \frac{b_{m_b} z^{m_b} + \cdots + b_1 z + b_0}{a_{m_a} z^{m_a} + \cdots + a_1 z + a x_0} \cdot z^{-d} \]  

The system with dynamic feedback is then

\[ G_z = \frac{G_R(z) G_S(z)}{1 + G_R(z) G_S(z)} = \frac{Q_z B_z}{R_z A_z + Q_z B_z} \]  

The final transfer function can be expressed as

\[ G(z) = \frac{\beta_\nu z^\nu + \cdots + \beta_1 z}{a_k z^k + a_{k-1} z^{k-1} + \cdots + a_0} \]  

The identification will yield estimation of coefficients of the closed loop system \( \beta_i \) and \( a_i \).

The order of the polynomial in the numerator in (1.4) is

\[ r = \nu + m_b + d \]  

and in the denominator

\[ k = \max[m_a, \nu + m_b + d] \]
To find the coefficients of the open-loop system we just need to compare coefficients of transfer functions (1.4) and (1.3). The system of linear equations can be written into matrix

\[
\begin{bmatrix}
q_\nu & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
q_{\nu-1} & q_\nu & \cdots & 0 & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
q_\nu & q_{\nu-1} & \cdots & q_0 & 0 & 0 & \cdots & 0 \\
0 & q_\nu & \cdots & 0 & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & q_\nu & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
q_\nu & q_{\nu-1} & \cdots & 0 & r_\mu & r_{\mu-1} & \cdots & r_0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
q_\nu & q_{\nu-1} & \cdots & q_0 & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & \cdots & q_\nu & 0 & 0 & \cdots & 0 \\
\end{bmatrix}
\begin{bmatrix}
b_1 \\
b_2 \\
\vdots \\
b_{\nu+1} \\
b_{\nu+2} \\
\vdots \\
b_{mb} \\
\end{bmatrix}
= 
\begin{bmatrix}
\beta_1 \\
\beta_2 \\
\vdots \\
\beta_{\nu+1} \\
\beta_{\nu+2} \\
\vdots \\
\beta_{mb} \\
\end{bmatrix}
\] (1.7)

We will get the sought coefficients by solving the linear system of equations (1.7). The solution can be written as a vector of parameters

\[
\hat{\theta} = [b_{mb} \cdots b_1 | a_{ma} \cdots a_1]^T
\] (1.8)

**Direct method** with noise source at the output is the other one to identify coefficients of forward system from closed loop system. The input value W(z) will be zero and the system will be excited by the disturbance Z(z), see figure (1.1).

![Figure 1.1: The arrangement of experiment for direct identification](image)

The order of polynomials of controller must meet requirements

\[
\max[\mu + m_a, \nu + m_b] > m_a + m_b
\] (1.9)
\[ \max[\mu - m_b, \nu - m_a] > 0 \]  
(1.10)

Keeping these requirements will cancel the correlation between error signal \( z(t) \) and signal going into the system \( u(t) \). The transfer function of the closed loop system can be expressed as

\[
\frac{Y(z)}{Z(z)} = \frac{G_p}{1 + G_R G_S} = \frac{D(z)R(z)}{A(z)R(z) + B(z)Q(z)}
\]  
(1.11)

which can be rewritten as

\[
A(z)R(z)Y(z) + B(z)Q(z)Y(z) = D(z)R(z)Z(z)
\]  
(1.12)

For desired value \( \omega = 0 \) is the control deflection \( e(t) = -y(t) \). The transfer function of the regulator will be then

\[
\frac{U(z)}{Y(z)} = -\frac{Q(z)}{R(z)}
\]  
(1.13)

Inserting (1.13) into (1.12) we will get

\[
A(z)R(z)Y(z) - B(z)R(z)U(z) = D(z)R(z)Z(z)
\]  
(1.14)

and dividing by \( R(z) \) will yield model ARMAX for open-loop system. From this we can see that is is possible to estimate parameters of the model directly with disturbance at the output and \( W(z) = 0 \).

**Solution in Matlab**

**Longitudinal Motion**

For handling qualities prediction, the military standard places requirements on the short period damping ratio \( \zeta_{sp} \), and the product of the short period natural frequency and inverse of the high frequency pitch attitude zero \( \omega_{sp}T_{\theta_2} \), which can be obtained from the pitch rate and normal acceleration LOES model

\[
\ddot{q} = \frac{K_\theta}{s^2 + 2\zeta_{sp}\omega_{sp}s + \omega_{sp}^2}
\]  
(1.15)

Inserting the parameters of our model will yield

\[
\dot{\bar{q}} = \frac{1.239(s + 0.404)}{s^2 + 0.9612s + 1.823}
\]

Discretization of the model e.g.

\[
c2d(sys_{sp}, 0.05, 'zoh')
\]

\[
G_S(z) = \frac{0.06108z - 0.05985}{z^2 - 1.949z + 0.9531}
\]

and

\[
G_R(z) = \frac{0.1z - 0.05}{z - 1}
\]

The experiment arrangement is on figure (B.1). The settings of the generator are \( T_s = 0.1 \), \( Seed = 23341 \), \( Np = 2.5 \). The identification is not depended on magnitude of
the input ($N_p$ can be varied), and on the dynamic of input. Damping ratio in the transfer function of dynamic input can be varied as well. The following program creates matrix (1.7) and computes parameters (1.8) of the system $G_s$. Comparing the results to the model in figure (B.1) we will learn that this method provides satisfactory results.

```
th = arx([[y,u],[3 3 1 ]]);
q = [0.1 -0.05 ];
r =[1 -1];
[dz nz] = th2poly(th)

% dz = [1 a1 a2 a3], dz1 = [a1 a2 a3]
for i = 2: length(dz),
dz1(i-1) = dz(i);
end

% nz = [0 b1 b2 b3], nz1 = [b1 b2 b3]
for i = 2: length(nz),
nz1(i-1) = nz(i);
end

% $\alpha_1 - r_1$
dz1(1) = dz1(1) - r(2);

[res] = mysolve(q,r,dz1,nz1,2,2)
```

**where function mysolve is**

```
function [res] = mysolve(q,r,alpha,beta,sa,sb,k)

sbeta = length(beta);
salpha = length(alpha);
a = hank(q,sb,sbeta);
c = hank(r,sa,salpha);
b = zeros(size(c));
X = [a b k*a c]
Y = [beta alpha];
Y = Y';
res = X\Y;
```

Matrice $X$ corresponds to (1.7) and the submatrices $a, c$ are created by function hank¹

```
function [X] = hank(q,m1,n);
while size(q) < n, 
q = [q 0];
end
for j = 1:n,
for i = 1:m1,
if (j-i+1) > 0,
X(j,i) = q(j-i+1);
end
```

¹Type help hank, help mysolve in Matlab
The estimated coefficients are

\[ \hat{\theta} = \begin{bmatrix} 0.0611 & -0.0598 & -1.9490 & 0.9531 \end{bmatrix}^T \]

The agreement of the estimated parameters and original ones is obvious. Let’s consider 4\textsuperscript{th} order system with phugoid poles. The transfer function is

\[ G_S(z) = \frac{-0.06108z^3 + 0.182z^2 - 0.1807z + 0.0598}{z^4 - 3.948z^3 + 5.848z^2 - 3.852z + 0.9523} \]

and controller

\[ G_R(z) = \frac{z + 0.08}{z^2 + 0.21z + 0.1} \]

Using the same procedure we will get a result

\[ \hat{G}_S(z) = \frac{-8.8305e - 05z^3 - 0.059154z^2 + 0.17799z - 0.18322}{z^4 - 3.9476z^3 + 5.8444z^2 - 3.8446z + 0.96607} \]

We can see the result is very closed to the original model only zeros differ. The reason is that the phugoid poles of the original model are very closed to the right plane and it is difficult to stabilize the system. As mentioned in chapter three, the results depend on the input dynamic.

In the other case we will assume zero input which can imagine zero deflection of elevator and disturbance at the output. The experiment arrangement is on figure (B.2). The accuracy of identification depends very on the structure of the controller and on the sample time of the error source, which is \textit{band limited white noise} in our case. The settings of the block is \textit{Np = 0.5, Ts = 0.2, Seed = 23341}. The procedure is as following:

\begin{verbatim}
  th = armax([y1 u1],[2 2 0 1]);
  compare([y1 u1],th)
  present(th)
  [dz nz] = th2poly(th);
  printsys(nz,dz,'z')
\end{verbatim}

The result differs significantly from the original model.

\[ \hat{G}_S(z) = \frac{-0.0276z + 0.12221}{z^2 - 0.88039z + 0.13938} \]

Comparison of the measured and estimated model for both methods is in appendix (C).

**Lateral Motion**

As mentioned above, for the lateral modes, the military standard requires only 3 parameters to be estimated: the roll-mode time constant \((T_{R})\), the Dutch roll damping ratio \((\zeta_d)\), and the Dutch roll natural frequency \((\omega_d)\). These parameters can be obtained e.g. from

\[ \frac{\ddot{\beta}}{da} = \frac{A_\beta \left( s + \frac{1}{T_{\Omega_1}} \right) \left( s + \frac{1}{T_{\Omega_2}} \right)}{\left( s + \frac{1}{T_{\beta}} \right) \left( s + \frac{1}{T_R} \right) \left( s^2 + 2\zeta_d\omega_ds + \omega_d^2 \right)} \]  

(1.16)
Though this would yield all of the parameters required for handling qualities prediction, the model contains a substantial number of parameters which would be difficult to estimate accurately. Our main interest is the denominator and this is always easier to identify than zeros. Evaluating (1.16) will yield

\[ G_S(z) = \frac{-1.892e - 05z^3 + 1.742e - 05z^2 + 1.935e - 05z - 1.785e - 05}{z^4 - 3.963z^3 + 5.892z^2 - 3.894z + 0.9656} \]

The controller is

\[ G_R(z) = \frac{z - 0.08}{z^2 + 0.21z + 0.1} \]

The result of the identification does not depend on any settings of the input generator and it is nearly the same as the original model apart from numerator which is not important for predicting the handling qualities. Using the same procedure, we will obtain

\[ G_S(z) = \frac{3.1925e - 07z^3 - 2.7597e - 05z^2 + 3.7747e - 05z + 4.6194e - 05}{z^4 - 3.963z^3 + 5.8918z^2 - 3.8938z + 0.96568} \]

All the results for both longitudinal and lateral motion are in appendix (A).

Summary

Two methods, indirect and direct were tried. Indirect method came out to be very accurate and resistant to any change of the input signal from its dynamic. The estimated parameters were nearly the same as those of the original model. The best results were obtained when the input signal was brought to the elevator in case of longitudinal motion and angle of attack was recorded. In lateral motion, the best result was obtained for relation rudder - roll rate. The hankel matrix was used to compute the system from the close loop.
Bibliography


Appendix A

Results of Close Loop Identification

Input Signals

![Diagram](image)

Figure A.1: Generator of input signal

**Settings:** Sample time = 0.1s, Power = 1, Seed = [23341]

\[
Filter_1 = \frac{0.01087s + 1}{2.717e - 005s^3 + 0.00413s^2 + 0.12s + 2.1}
\]

\[
Filter_2 = \frac{0.01087s + 1}{2.717e - 005s^3 + 0.00413s^2 + 0.25s + 2.1}
\]

\[
Filter_3 = \frac{0.01087s + 1}{2.717e - 005s^3 + 0.0035s^2 + 2.25s + 2.1}
\]
Longitudinal Motion

Input: elevator - $de$
Output: angle of attack - $\alpha$

Controller:

$$G_R(z) = \frac{2.1793e - 06}{z^2 - 1.94z + 0.9414}$$

$k = 0.00131$

System transfer function:

$$G_S(z) = \frac{-1.326z^3 + 1.347z^2 + 1.28z - 1.301}{z^4 - 3.948z^3 + 5.848z^2 - 3.852z + 0.9523}$$

Identified system:

$$\hat{G}_S(z) = \frac{-1.3257z^3 + 1.3473z^2 + 1.2796z - 1.3012}{z^4 - 3.9478z^3 + 5.8478z^2 - 3.8523z + 0.95226}$$

![Figure A.2: Step Response](image)

### Table A.1: Parameter Values and Flying Qualities Predictions

<table>
<thead>
<tr>
<th>$\zeta_{SP}$</th>
<th>$\omega_{SP}$</th>
<th>$T_{a_2}$</th>
<th>Category</th>
<th>Level</th>
<th>Reason for Not Level 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>1.59</td>
<td>1.7362</td>
<td>B</td>
<td>2</td>
<td>$\zeta_{SP} &lt; 0.3$</td>
</tr>
</tbody>
</table>
Lateral Motion

Input: rudder - $dr$
Output: roll rate - $p$

Controller:

$$G_R(z) = \frac{0.068936(z^2 - 2.222z + 1.235)}{(z^2 - 1.738z + 0.7597)}$$

$k = 0.0417$

System transfer function:

$$G_S(z) = \frac{-0.004744z^3 - 0.001796z^2 + 0.01779z - 0.01125}{z^4 - 3.963z^3 + 5.892z^2 - 3.894z + 0.9656}$$

Identified system:

$$\hat{G}_S(z) = \frac{-0.004744z^3 - 0.0017958z^2 + 0.01779z - 0.01125}{z^4 - 3.963z^3 + 5.8916z^2 - 3.8943z + 0.96565}$$

From: rudder
To: roll rate

Figure A.3: Step Response

<table>
<thead>
<tr>
<th>$\zeta_d$</th>
<th>$\zeta_d\omega_d$</th>
<th>$\hat{T}_R$</th>
<th>Category</th>
<th>Level</th>
<th>Reason for Not Level 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.168</td>
<td>0.1651</td>
<td>2.6738</td>
<td>B</td>
<td>2</td>
<td>$\hat{T}_R &gt; 1.4$</td>
</tr>
</tbody>
</table>
Appendix B

Simulink models

Figure B.1: Closed loop system - indirect method

Figure B.2: Closed loop system - direct method
## Appendix C

### List of Files for Matlab 6.0

#### Matlab Files

<table>
<thead>
<tr>
<th>File</th>
<th>Description</th>
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<tbody>
<tr>
<td>hank</td>
<td>Compute hankel matrix</td>
</tr>
<tr>
<td>ident_fb1</td>
<td>Identification of longitudinal motion from closed loop</td>
</tr>
<tr>
<td>ident_fb2</td>
<td>Identification of lateral motion from closed loop</td>
</tr>
<tr>
<td>init_fb1</td>
<td>Initialization of Boeing 747-400, longitudinal motion</td>
</tr>
<tr>
<td>init_fb2</td>
<td>Initialization of Boeing 747-400, lateral motion</td>
</tr>
<tr>
<td>mysolve</td>
<td>Indirect method of parameter estimation from closed loop</td>
</tr>
<tr>
<td>setzero</td>
<td>Set negligible values to zero</td>
</tr>
<tr>
<td>th_</td>
<td>Identify system by ARX method</td>
</tr>
<tr>
<td>th_m</td>
<td>Identify system by ARMAX method</td>
</tr>
<tr>
<td>th_oe</td>
<td>Identify system by Output Error method</td>
</tr>
<tr>
<td>thd</td>
<td>Identify the high frequency pitch attitude zero</td>
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<tr>
<td>zero_identf</td>
<td>Display high frequency pitch attitude zeros for all experiments</td>
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<td>mdf1.mdl</td>
<td>Experiments in closed loop system for longitudinal motion</td>
</tr>
<tr>
<td>mdf2.mdl</td>
<td>Experiments in closed loop system for lateral motion</td>
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